Math 31 - Homework 2

Due Wednesday, July 3

Easy

1. [Saracino, Section 2, #1 (a), (b), (h), (i)] Which of the following are groups? Why? (That is, either verify that the axioms hold, or explain why one of them fails.)

(a) $\mathbb{R}^+$ under addition. (Here $\mathbb{R}^+$ denotes the set of all *positive* real numbers.)

(b) The set $3\mathbb{Z}$ of integers that are multiples of 3, under addition.

(c) $\mathbb{R} - \{1\}$ under the operation $a * b = a + b - ab$.

(d) $\mathbb{Z}$ under the operation $a * b = a + b - 1$.

2. [Saracino, Section 2, #5] The following table defines a binary operation on the set $S = \{a, b, c\}$.

$*$	a	b	c
a	a	b	c
b	b	b	c
c	c	c	c

Is $\langle S, * \rangle$ a group?

Medium

3. [Saracino, Section 2, #8] Let G be the set of all real-valued functions f on the real line which have the property that $f(x) \neq 0$ for all $x \in \mathbb{R}$. In other words,

$$G = \{f : \mathbb{R} \to \mathbb{R} : f(x) \neq 0 \text{ for all } x \in \mathbb{R}\}.$$

Define the product $f \times g$ of two functions $f, g \in G$ by

$$(f \times g)(x) = f(x)g(x) \text{ for all } x \in \mathbb{R}.$$

With this operation, does G form a group? Prove or disprove.

4. If G is a group in which $a * a = e$ for all $a \in G$, show that G is abelian.

Extra credit: We saw in class that any group of order 1, 2, or 3 is abelian. Show that any group of order 4 must be abelian. [Hint: Try to write down all the possible group tables in this case. Up to reordering the elements of the group, there will only be two possibilities.]

Extra extra credit: Try to extend the first extra credit problem by showing that any group of order 5 must also be abelian.